A Robust Optimisation Strategy for Metal Forming Processes

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Abstract. Robustness, reliability, optimisation and Finite Element simulations are of major importance to improve product quality and reduce costs in the metal forming industry. In this paper, we propose a robust optimisation strategy for metal forming processes. The importance of including robustness during optimisation is demonstrated by applying the robust optimisation strategy to an analytical test function and an industrial hydroforming process, and comparing it to deterministic optimisation methods. Applying the robust optimisation strategy significantly reduces the scrap rate for both the analytical test function and the hydroforming process.

Keywords: optimisation, metal forming, FEM, robustness, reliability

INTRODUCTION

Product improvement and cost saving have always been important goals in the metal forming industry. One way of achieving these two goals is optimising towards robust metal forming processes. A robust metal forming process will yield metal products at a more constant quality level. Hence, it will (i) improve the product's quality; and (ii) save costs because the number of non-feasible products (scrap) is decreased. Generally, optimisation strategies only include deterministic control variables. To assess the robustness of a metal forming process, the noise variables (e.g. material variation) need to be taken into account during optimisation.

In [1], we presented three ways to optimise towards robust metal forming processes using time consuming Finite Element simulations of these processes: deterministic optimisation, robust optimisation and reliability based optimisation. In [2], robust optimisation techniques have been further developed to yield a robust optimisation strategy for metal forming processes. In this paper, the robust optimisation strategy will be summarised and compared to deterministic optimisation by application to an analytical test function. Both the deterministic and robust optimisation strategy will be applied to an industrial hydroforming example.

A ROBUST OPTIMISATION STRATEGY FOR METAL FORMING PROCESSES

The proposed robust optimisation strategy is an extension of a deterministic optimisation strategy for metal forming processes presented in [3]. This strategy consists of three stages: a structured methodology for modelling optimisation problems in metal forming, screening techniques to reduce the problem size (i.e. the number of design variables), and a Sequential Approximate Optimisation (SAO) algorithm for solving the problem. A flowchart of the optimisation strategy is presented in Figure 1(a). The robust optimisation strategy differs from the deterministic strategy in the modelling, optimisation and evaluation parts.

Concerning the modelling, noise variables are included in addition to deterministic control variables. For the noise variables, a normal distribution is assumed. For each response (objective function or constraint), one now obtains a response distribution (μ_y and σ_y) instead of a response value y. As objective function f one can optimise μ_f, σ_f or a weighted sum $\mu_f \pm w\sigma_f$. If μ_f or σ_f are optimised, it is advised to include the weighted sum as a constraint: this takes into account process reliability in the optimisation problem. Also other constraints g are taken into account as a weighted sum $\mu_g \pm w\sigma_g$.

Figures 1(b) and (c) compare the differences in the optimisation algorithms and optimum evaluation for the deterministic and robust optimisation strategies. The difference in optimisation is the determination of the separate metamodels for μ_y and σ_y. For this, we employ a Single Response Surface technique, which fits one metamodel in both the control and noise design variable space, e.g. the following RSM metamodel which is quadratic in the design variable space and linear + interaction in the noise variable space:

$$\hat{y}(\mathbf{x},\mathbf{z}) = \beta_0 + \mathbf{x}^{\mathrm{T}}\beta + \mathbf{x}^{\mathrm{T}}\mathbf{B}\mathbf{x} + \mathbf{z}^{\mathrm{T}}\gamma + \mathbf{x}^{\mathrm{T}}\Delta\mathbf{z} + \varepsilon \quad (1)$$

where $\hat{y}$ is a single metamodel of a response dependent on the control variables $\mathbf{x}$ and noise variables $\mathbf{z}$. β_0, β, $\mathbf{B}$, γ and Δ denote the fitted regression coefficients and ε is the random error term. From Equation 1, one can analytically determine two RSM metamodels for mean and variance [4]:

$$\begin{aligned} \mu_y &= E[\hat{y}(\mathbf{x},\mathbf{z})] = \beta_0 + \mathbf{x}^{\mathrm{T}}\beta + \mathbf{x}^{\mathrm{T}}\mathbf{B}\mathbf{x} \\ \sigma_y^2 &= \mathrm{var}[\hat{y}(\mathbf{x},\mathbf{z})] = \sigma_z^2(\gamma^{\mathrm{T}} + \mathbf{x}^{\mathrm{T}}\Delta)(\gamma + \Delta^{\mathrm{T}}\mathbf{x}) + \sigma^2 \end{aligned} \quad (2)$$

CP908, *NUMIFORM '07, Materials Processing and Design: Modeling, Simulation and Applications*
edited by J. M. A. César de Sá and A. D. Santos

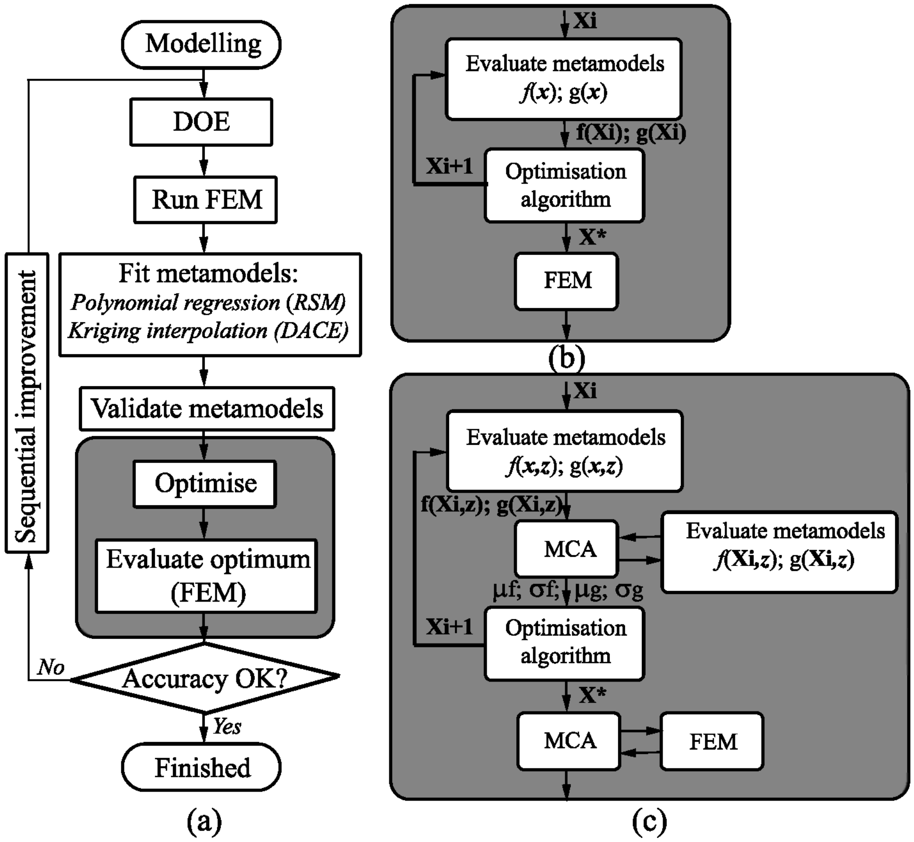

FIGURE 1. (a) Flow chart of the optimisation strategy; (b) Deterministic; (c) Robust

with μ_y and σ_y^2 the metamodels for mean and variance of the response.

When Kriging is employed instead of RSM, an analytical derivation of μ_y and σ_y is not possible. In this case we run a Monte Carlo Analysis (MCA) on the fitted metamodel as shown in Figure 1(c). Single Response Surface techniques are a relatively efficient way of robust optimisation [2].

The difference between robust and deterministic optimisation (see Figure 1) in the evaluation of the optimum $\mathbf{X}^*$ is that, in the deterministic case, this can be done by running one final FEM calculation. In case the robustness and reliability need to be assessed after optimisation, it is necessary to run an MCA using FEM calculations, which is quite time consuming.

APPLICATION TO AN ANALYTICAL TEST FUNCTION

The robust optimisation strategy will be compared to the deterministic optimisation strategy by application to the analytical test function presented in Figure 2(a). Figure 2(b) presents the contour of this objective function as well as a constraint. The constrained deterministic optimisation problem is:

$$\begin{aligned} \min f &= 12 + x_1^2 + \frac{1+x_2^2}{x_1^2} + \frac{x_1^2 x_2^2 + 100}{(x_1 x_2)^4} \\ \text{s.t. } g &= 6.5 - x_2 \leq 0 \\ & 0.1 \leq x_1, x_2 \leq 10 \end{aligned} \tag{3}$$

For the unconstrained deterministic optimisation model, the constraint g is simply omitted. Both the unconstrained and constrained deterministic optima are presented in Figure 2(b).

The robust optimisation problem is modelled as:

$$\begin{aligned} & \min \mu_f \\ & \text{s.t. } \mu_f + 3\sigma_f \leq 50 \\ & \quad \mu_g + 3\sigma_g \leq 0 \\ & \quad 1 \leq x_1, x_2 \sim N(\mu, 0.4) \leq 10 \end{aligned} \tag{4}$$

Again the unconstrained (g omitted) and the constrained problem have been optimised, this time using the robust optimisation strategy. 100 function evaluations are run

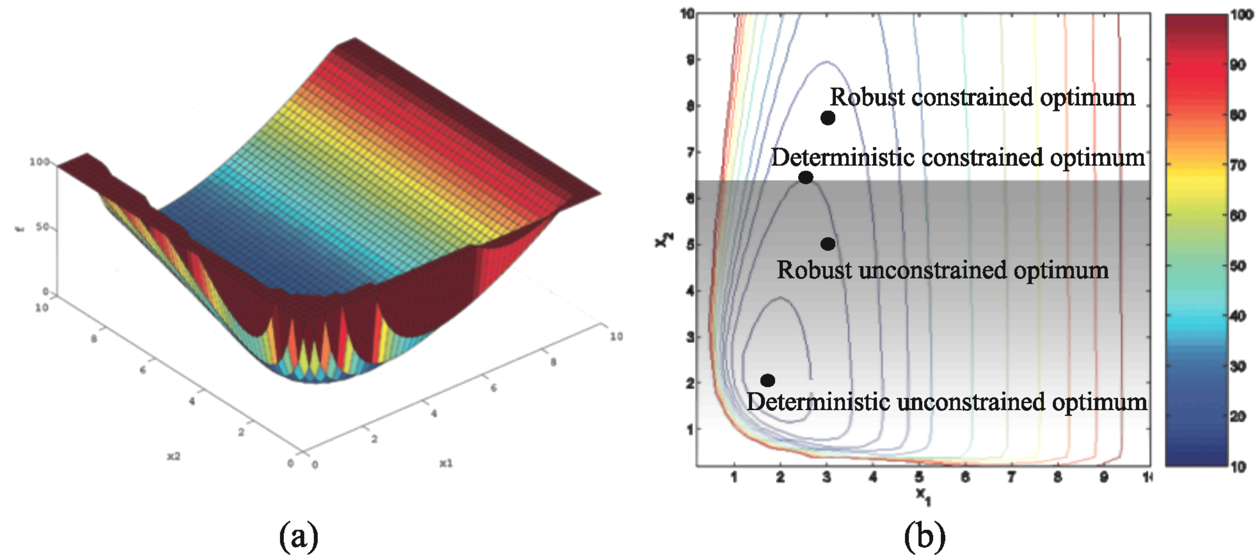

FIGURE 2. (a) Analytical test function; (b) Contour plot including optima

for each optimisation. Both corresponding optima are again displayed in Figure 2(b).

After optimisation, the reliability of all optima has been evaluated using an MCA of 20000 function evaluations. Figure 3 compares the results of deterministic and robust unconstrained optimisation. The scrap rate has been reduced from 0.92% for the deterministic optimum to << 0.005% for the robust optimum.

The improvement of the robust optimisation strategy w.r.t. the deterministic one is even much more dramatic in constrained cases as depicted in Figure 4. For the deterministic optimum, the scrap rate due to violation of the constraint g is 50.3% (Figure 4(b)). For the robust optimum, Figure 4(d) shows that the scrap rate has been reduced to 0.1%, which nicely corresponds to the 3σ reliability level modelled in Equation 4.

APPLICATION TO HYDROFORMING

In this section, both the deterministic and robust optimisation strategies will be applied to an industrial hydroforming process, which demonstrates that both strategies are indeed applicable to optimise metal forming processes using time consuming Finite Element simulations. The concerned part is a hydroformed car bumper designed by Corus. Half of the part is shown in Figure 5. AutoForm is used as FEM code, calculations take about 75 minutes per simulation.

The deterministic optimisation strategy has been used to model the optimisation problem. Screening techniques have been employed to reduce the number of design variables to the two most important ones. The reduced optimisation model remaining after screening is:

$$\begin{aligned} \min f &= \max_N \frac{\varepsilon_1(\varepsilon_2)}{\varepsilon_1^{\mathrm{flc}}(\varepsilon_2)} - 1 \\ \text{s.t. } g &= d_{\mathrm{filling}} - 3 \leq 0 \\ 56 &\leq x_1 = R \leq 60 \text{ mm} \\ 50 &\leq x_2 = p_3 \leq 300 \text{ MPa} \end{aligned} \quad (5)$$

where the objective function aims to overcome necking by maximising the distance of the major strains to the Forming Limit Curve (FLC) while making sure that the final part fills out the tools nicely (the gap between final product and tool should not exceed 3 mm). The two most important design variables found using screening are the radius of the initial tube R and the fill out pressure p_3.

Applying the deterministic SAO algorithm from Figure 1, the optimum has been found after 53 FEM simulations. The results in Table 1 present that a 31% margin below the FLC has been reached, while a negative value for the constraint g denotes the filling of the product satisfies the demands.

However, it is well-known that material parameters such as the R-values display variation. This input variation is transferred to the objective function and constraint. One can check the robustness and reliability of the obtained deterministic optimum by running a Monte Carlo Analysis (MCA). Figures 6(a) and (b) present the response histograms of a 200 FEM run MCA for both f and g, respectively. As shown in Table 1, the scrap rate is 41.2%, which is totally due to violation of the filling constraint g.

Let us now see whether the robust optimisation algorithm is able to reduce this scrap rate. Following the ro-

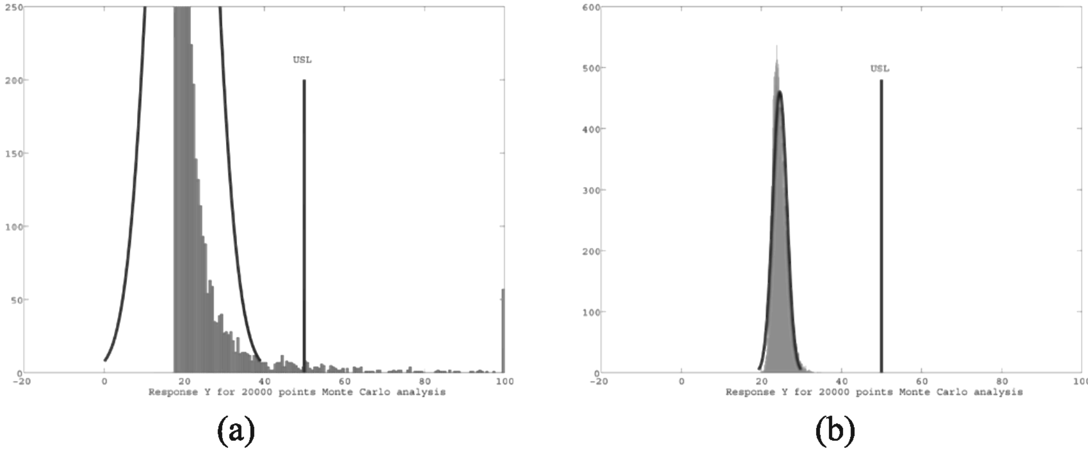

FIGURE 3. Response distributions: (a) Deterministic unconstrained optimum; (b) Robust unconstrained optimum

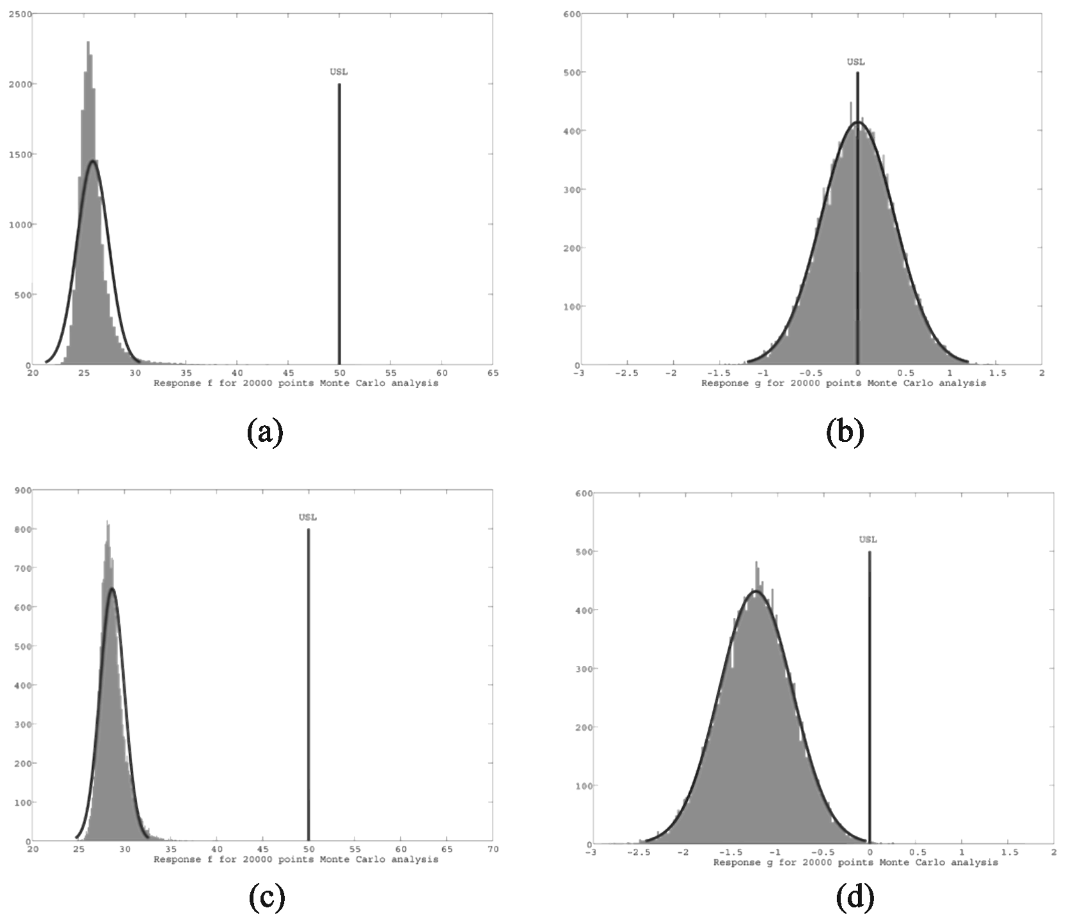

FIGURE 4. (a) Objective function distribution deterministic constrained optimum; (b) Constraint distribution deterministic constrained optimum; (c) Objective function distribution robust constrained optimum; (d) Constraint distribution robust constrained optimum

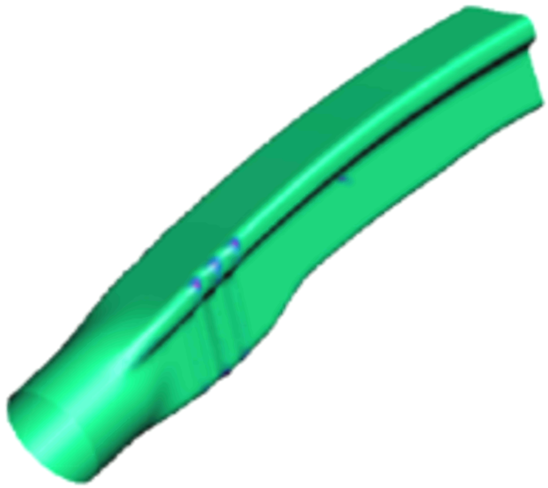

FIGURE 5. Industrial hydroforming part (Corus design)

bust optimisation strategy, the robust optimisation problem becomes:

$$
\begin{aligned}
&\min \mu_f \\
&\quad \text{s.t. } \mu_f + 3\sigma_f \leq 0 \\
&\qquad \mu_g + 3\sigma_g \leq 0 \\
&\qquad 56 \leq x_1 = R \leq 60 \text{ mm} \\
&\qquad 50 \leq x_2 = p_3 \leq 300 \text{ MPa} \\
&\qquad \mu_z - 4\sigma_z \leq R_0, R_{45}, R_{90} \sim N(\mu_z, \sigma_z) \leq \mu_z + 4\sigma_z
\end{aligned}
\tag{6}
$$

where R_0, R_{45} and R_{90} denote the material's R-values in the rolling, 45 degrees and transverse directions. Hence, it is tried to minimise the mean value of the objective function while putting a 3σ reliability demand on both the objective function and constraint.

The robust optimisation algorithm has been applied to solve the optimisation problem in Equation 6. 200 FEM simulations have been run in the 5D combined control-noise variable space and a Kriging metamodel has been fitted and optimised. A 200 FEM analysis MCA was used to validate the optimum: its results are presented in Table 1, the response histograms are included in the Figures 6(c) and (d).

Although the scrap rate has been reduced to 27.9%, the table and the figures show that the constraint $\mu_g + 3\sigma_g \leq 0$ is not satisfied. This may be due to inaccuracy of the metamodel. Just as was the case for the analytical test function in the previous section, an accurate metamodel would have reduced the scrap rate further, if possible to the required 3σ level, i.e. a scrap rate of maximum 0.3%.

CONCLUSIONS AND FUTURE WORK

Robustness, reliability, optimisation and Finite Element simulations are of major importance to improve product quality and reduce costs in the metal forming industry. In this paper, we proposed a robust optimisation strategy for metal forming processes. In addition to deterministic control variables, the strategy explicitly takes into account noise variables such as material variation and optimises probability distributions of objective function and constraints in order to achieve a robust and reliable metal forming process. The importance of including robustness during optimisation has been demonstrated by applying the robust optimisation strategy to an analytical test function: for constrained cases, deterministic optimisation will yield a scrap rate of about 50% whereas the robust optimisation strategy reduced this scrap rate to the demanded 3σ reliability level. The strategy has also been applied to an industrial hydroforming process. Applying the robust strategy above the deterministic one also reduced the scrap rate in this case. Although the scrap rate has not been reduced as much as required, the application to hydroforming underlines the potential of the developed robust optimisation strategy for robustly optimising industrial metal forming processes using time consuming FEM simulations.

Future work comprises improving the metamodel – at least in the vicinity of the optimum – by developing sequential improvement strategies for the robust algorithm (see Figure 1).

ACKNOWLEDGMENTS

This research has been carried out in the framework of the project "Optimisation of Forming Processes MC1.03162", which is part of the research programme of the Netherlands Institute for Metals Research (NIMR). The industrial partners co-operating in this project are gratefully acknowledged.

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TABLE 1. Results of optimising the hydroforming process

	R	p_3	f	g	**Scrap rate**
Deterministic optimum	58.1	239	-0.313	-0.16	41.2%
Robust optimum	59.3	258	$\mu = -0.234$	$\mu = -0.659$	27.9%
			$\sigma = 0.045$	$\sigma = 0.668$	

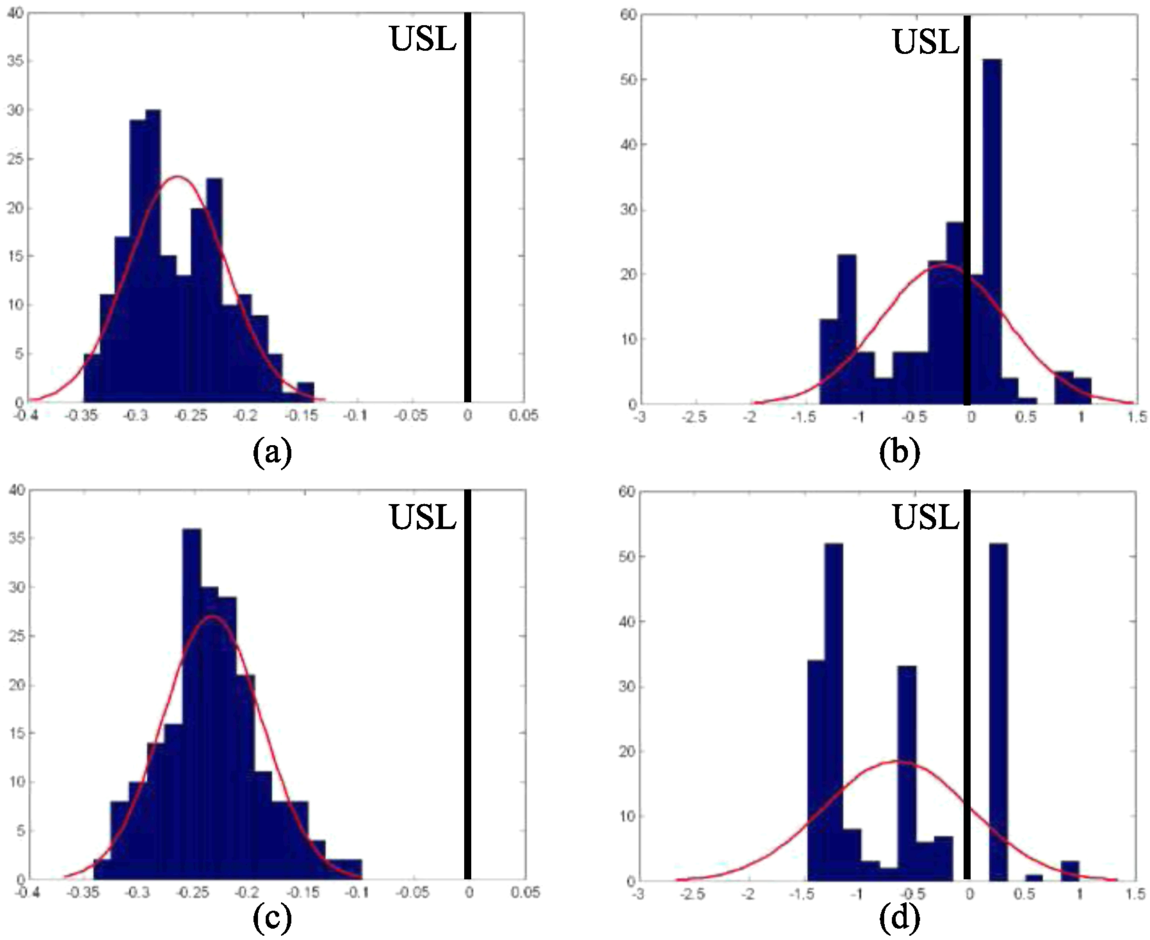

FIGURE 6. Monte Carlo Analysis of the hydroforming example: (a) Deterministic optimum f; (b) Deterministic optimum g; (c) Robust optimum f; (d) Robust optimum g